

THE USE OF THE REPRESENTATIVENESS AND AVAILABILITY HEURISTIC IN PROBABILISTIC REASONING BY STUDENTS IN THE GRADUATE DIPLOMA OF EDUCATION QUT, 1992

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BACKGROUND

"Probability as the basis for decision making" is a topic of the new Queensland Senior Maths Syllabus. This is as part of what is referred to as "mathematics for intelligent citizenship" and is core for all students.

Misconceptions in probabilistic reasoning are widely reported in the literature. Tversky and Kahneman (1982) report on the use of "representativeness" and "availability" by educated professional people. Shaughnessy (1977, 1981) reports on the use of the same heuristics by university students in the USA and Peard (1991) cites the occurrence of these in secondary school students in Australia. Scholz (1987) examines the misuse of representativeness by a large cross section of German society. He includes an extensive examination of what is referred to as "The base-rate fallacy". Bon-Hillel (1982) reports on follow-up studies to Tversky and Kahneman.

Other misconceptions in probabilistic reasoning include the occurrence of "counter intuitive results" (Peard 1991), misconceptions of regression and illusory correlation (Tversky and Kahneman, 1982).

It is the contention of the author that of all these misconceptions, representativeness and availability appear to be the most widely spread.

Shaughnessy (1981) defines representativeness. This term is used by "those who estimate the likelihood of an event on the basis of how similar the event is to the population from which it is drawn" (p.91). This results in a fallacy by which likely sequences are viewed as unequally likely. This can occur in two ways. For example, a subject observes a "fair" coin land heads on eight out of ten tosses, and then incorrectly reasons that the likelihood of a head on the next toss is less than one half. The subject is reasoning that a balance is expected (i.e. representative) even in the short run when in fact no such balance is expected. This is commonly known as "belief in the gambler's fallacy of the law of averages" (del Mas & Bart, 1989).

In the second instance, the subject erroneously reasons that a head is more likely using the preceding short run of outcomes as being a representative base rate.

Tversky and Kahneman (1982) submit that:

people view a sample randomly drawn from a population as highly representative, that is similar to the population in all essential characteristics. Subjects act as if every segment of the random sequence must reflect the true proportion: if the sequence has strayed from the population proportion, a

corrective bias in the other direction is expected. This has been called the gambler's fallacy. The heart of the gambler's fallacy is a misconception of the fairness of the laws of chance. The gambler feels that the fairness of the coin entitles him to expect that any deviation in one direction will soon be cancelled by a corresponding deviation in the other.

Whereas, in reality,

deviations are not cancelled as sampling proceeds, they are merely diluted. (p. 238)

They refer to this as the belief in the law of small numbers. The publication of lotto number frequencies of occurrence in Australian newspapers is evidence of the widespread nature of this misconception. Shaughnessy (1981) refers to availability as happening:

When people tend to make predictions about the likelihood of an event based on the ease with which instances of that event can be constructed, or called to mind, they are relying on availability. For example, if a person is asked to estimate the local divorce rate or to estimate the probability of being involved in an automobile accident, the frequency of his or her personal contact with these events (perhaps through friends or relatives) may influence the probability estimates. One who has recently been involved in an accident might estimate a high probability for the accident rate. One who has many friends and acquaintances who are divorced has more instances of divorce "available" and may estimate a higher local divorce rate than someone who does not know many divorced people.

Availability causes systematic bias in probability estimates because people tend to believe that those outcomes that can easily be brought to mind will be more likely to occur.

For example, in buying prenumbered lottery tickets, if people tend to avoid numbers such as 123456 or 699999, it is because they cannot easily recall such numbers occurring, whereas they can recall "unpatterned" numbers readily and thus believe that any such number is more likely.

RESEARCH

This research examines the heuristic of representativeness and availability. The subjects were a sample of 24 students in the Graduate Diploma of Education (Mathematics and Science) at QUT, Semester 1, 1992. These all have some formal education in mathematics and most will presumably teach mathematics in Queensland schools - including the topics in probability.

A control group of 22 students was selected from The Graduate Diploma of Education in fields other than mathematics. These have no tertiary mathematics education.

The aims of the research were to determine:

1. how prevalent the misconceptions of representativeness and availability are amongst graduate students in education; and

2. whether or not our future mathematics teachers differ in the use of these heuristics from others.

Both groups of students were asked the same five questions (see Appendix). All were asked to attempt all questions, using intuitive knowledge if no probabilistic knowledge was available. The methods employed in Question 5 were discussed individually with approximately half of each group.

At the recent (May, 1992) QAMT annual conference, question 5 was asked a group of practising secondary mathematics teachers. Their responses are shown also.

ANALYSIS AND INTERPRETATION OF RESULTS

The proportion demonstrating the heuristics in a relatively simple situation (Question 1 and 2) is low. We further note that it is consistently lower for the Maths/Science group than for the others. However, statistical tests for significance of difference (Chi-squared) imply that further research using larger samples is required before any conclusions with a high degree of confidence can be made. Nevertheless, the results are interesting and show that Maths/Science graduates (at least in this sample) tend not to use either heuristic in these simple situations. The use by non-Maths/Science graduates is still relatively low when compared with reports in the literature stating the widespread use. It is further interesting to note that the usage by this group is similar to that of high school students reported in earlier research (Peard, 1991).

In Questions 3 and 4 in which the concepts are more "sophisticated" we see that there is no difference between the two groups. Further, the proportions are in general agreement with results obtained by Shaughnessy (1981) on identical questions asked to a sample of 80 first year college students in the USA. This is cause for some concern, since question 3 can be answered correctly by considering probability from combinations as is done in many senior secondary mathematics cases. These results tend to confirm "mere exposure to the theoretical laws of probability may not be significant to overcome misconceptions of probability." [Shaughnessey (1981) p. 91]

The results of Question 4 imply that the use of availability in this situation is very evident. Interviews with subjects showed that of seven answering correctly only three (all Maths/Science) were reasoning along the correct lines of a small sample being less likely to reflect the population.

Question 5 is the really interesting one. Here we see that roughly half of each group ignore the base rate. This is consistent with extension research by Scholtz (1987) who reported that the model response of all groups (professionals, graduates, non-graduates, lawyers, teachers, doctors and others) was "80%".

Of our group only one was correct and this one had a good background in statistics and recognised the question as a Bayesian conditional probability and solved it correctly. Six others, with some background in probability (all from the Maths/Science group) attempted a probabilistic solution and arrived at an incorrect (and quite obviously so) answer. Five employed the multiplication principle incorrectly $0.15 \times 0.8 = 0.12$ whilst one attempted to use Bayes' formula but arrived at an answer of 1%. On the other hand, the non-

Maths/Science graduates were more prepared to admit that they were not sure or had no idea, with nearly one third giving this response compared to none of the other group. I think that this has implications for our curriculum classes at university.

We note further that of the four who ignored the witness and relied only on the base rate, three were from the Maths/Science group. Of the six who recognised that the base rate does in fact reduce the probability from 80% (responses to 50 - 80 or <50), four were from the non-Maths/Science group.

Practising secondary teachers of mathematics fared little better, although now 8% correctly used a tree diagram to arrive at the correct answers. A smaller proportion (4%) used the multiplication principle incorrectly and 12% ignored the base rate. One responded >80% demonstrating a response that was more prevalent amongst Scholtz's subjects, in which the respondent's tendency is to "believe the witness".

CONCLUSIONS

I believe that the results have implications for university curriculum units. We must include and give particular attention to the teaching of probability. The concepts required to answer questions 3, 4, and 5 correctly are not beyond basic probability and well within what is required by most secondary mathematics syllabi. Nevertheless, our prospective secondary mathematics teachers are no better prepared to answer them correctly and fall back on the heuristics that are evident amongst the general population.

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QUESTION AND RESPONSES

1(a)	TOTAL	GRADUATES IN MATHS/SCIENCE	OTHER GRADUATES
<p>In the last 200 Lotto draws the number 17 has occurred more times than any other number. In future draws, is this number:</p>			
(i) more likely to occur than any other number?	4	0	4
(ii) less likely to occur than any other number?	3	1	2
(iii) equally likely to occur as any other number?	38	22	16
(iv) Not sure.	1	1	0
% demonstrating representativeness	17%	8%	27%
<p>1(b)</p> <p>A roulette wheel has 18 red and 18 black numbers. The probability of the ball landing on a red is the same as landing on a black. A gambler observes the ball to land on the following numbers in a row of six: RRRRRR. If he decides to be on the next throw is it more likely to be:</p>			
(i) red	2	1	1
(ii) black	7	1	6
(iii) red and black are equally likely	36	21	15
(iv) not sure	1	1	-
% demonstrating representativeness	20%	12%	32%
<p>1(c)</p> <p>If you could choose any of the following lottery tickets, in a lottery that has the numbers written on the ticket you buy, would you have any preference for:</p>			
(i) 123456	2	0	2
(ii) 619999	0	0	0
(iii) 615472	9	2	7
(iv) no preference for any of these	35	22	13
% demonstrating availability	26%	8%	32%

2	TOTAL	GRADUATES IN MATHS/SCIENCE	OTHER GRADUATES
(a) If I throw a single fair die, are the numbers one through six equally likely to occur?			
No	43	24	19
Yes	3	0	3
(availability) %	7	0	14
(b) If I throw a pair of dice, it is harder to get a pair of sixes than it is to get a pair of twos?			
No	43	24	19
Yes	3	0	3
(availability) %	7	0	14
(c) A poker hand consists of five cards randomly dealt from the deck of 52. Ace is the highest card. Is it harder (i.e. less likely) to get 3 Aces than it is to get three fives?			
No	41	23	18
Yes	5	1	4
(availability) %	11	4	18
(d) You and I play a game in which a coin is tossed. Heads I win. Tails you win. Of the last 15 people I played only 5 won. Is the game fair? Is the coin fair?			
Game Yes	38	22	16
No	8	2	6
Coin Yes	38	22	16
No	8	2	6
(representativeness) %	17	8	27
3			
A person must select committees from a group of 10 people. (A person may serve on more than one committee). Would there be			
(a) more distinct possible committees of 8 people?	6	1	5
(b) more distinct possible committees of 2 people?	21	12	9
(c) about the same number of committees of 8 as committees of 2?	16	10	6
(d) not sure.	3	1	2
% demonstrating availability (selecting (b))	46	50	41

	TOTAL	GRADUATES IN MATHS/SCIENCE	OTHER GRADUATES
4			
The chance that a baby will be a boy is about $\frac{1}{2}$. Over the course of an entire year, would there be more days when at least 60 percent of the babies born were boys -			
(a) in a large hospital	3	1	2
(b) in a small hospital	7	5	2
(c) makes no difference	33	17	16
(d) not sure	3	1	2
% demonstrating availability	72	71	73
5			
In a certain town there are two types of taxis; 15% are Blue cabs and 85% are Green cabs. A taxi is involved in a hit-run accident. A witness identifies it as a Blue cab. The court tests the witness' ability to distinguish between the two under identical conditions and concludes that the witness is right 80% of the time.			
Question: What is the probability that the cab was a Blue cab?			
	%		
	0	0	0
>80	22(48%)	12(50%)	10(45%)
80	4(9%)	1(4%)	3(14%)
50-80	2(4%)	1(4%)	1(5%)
<50	1(2%)	1(4%)	0
41*	4(8%)	3(13%)	1(5%)
15**	5(10%)	5(21%)	0
12*	1(2%)	1(4%)	0
<1	7(15%)	0	7(32%)
not sure/no idea			
Practising secondary mathematics teachers			
>80		1	
80			
50-80			
<50		3	
41			
15			
12			
<1			
not sure / no idea			